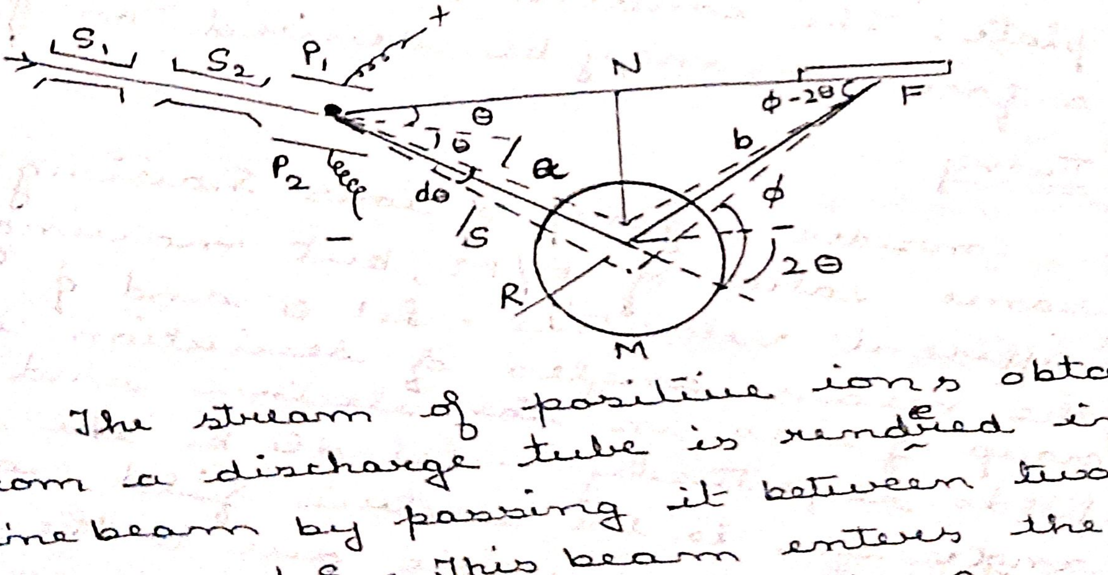
$$\frac{y}{x} = \frac{k_2 B}{k_1 X} v \quad \text{ie} \quad \boxed{\frac{y}{x} \propto v}$$

Thus the position of any individual particle on the parabola will depend on the velocity of the particle. Hence the entire parabola is a velocity dispersion or velocity spectrum.

Determination of E/M.

The values of E/M can be calculated from equation (3) by measuring the coordinates of x and y for a point on the parabola evaluating the constant k_1 and k_2 for the apparatus and knowing B and X.

Aston's Mass spectrograph.



The stream of positive ions obtained from a discharge tube is rendered into a fine beam by passing it between two narrow slits S_1 and S_2. This beam enters the electric field between the metal plates P_1 and P_2. Due to the action of the electric field (X), all positive ions having the same value of E/M are not only deviated by an angle θ from the original path but are dispersed by an angle $d\theta$ due to their different velocities. The beam is then allowed to pass through a magnetic field M acting at right angles to the electric field so that it produces a deflection of the beam in the same plane. T magnetic field deviates the particles by an angle ϕ and reconverges them by $d\phi$. The direction and magnitude of the field is so adjusted that it produces a deviation of the beam in the opposite direction and brings all ions having the same value of E/M, eventhough differing in velocities, To a focus at one point F, Ions having different values of E/M are brought to focus ~~to one point~~ at different points on the photographic plate. The condition required for such a focusing may be derived as follows.

Theory.

Consider a group of ions having the same value of E/M, but moving with different velocities. Let θ and ϕ be the mean angles of deviation of the group of ions in the electric and magnetic fields respectively. Let $d\theta$ be the dispersion angle due to the electric field and $d\phi$ the convergence angle due To the magnetic field.

If X - strength of electric field, E charge of ion
M. - mass of the ion, v - velocity of ion.
l_1 - length of electric field and d_1 the linear displacemt of an ion from its

path due to electric field.

$s = \frac{1}{2}at^2$

Then $d_1 = \frac{1}{2}\left(\frac{F}{m}\right)\left(\frac{l_1}{v}\right)^2$

$ma = F$

$a = \frac{F}{m} = \frac{XE}{m}$

$= \frac{1}{2}\left(\frac{XE}{m}\right)\left(\frac{l_1}{v}\right)^2$

$t = \frac{dist}{dt} = \frac{l_1}{v}$

$d_1 = \frac{1}{2}\frac{XE}{m}\frac{l_1^2}{v^2}$

angular deviation of beam $\theta = \frac{d_1}{l_1}$

$= \frac{\frac{1}{2}\frac{XE}{m}\frac{l_1^{\cancel{2}}}{v^2}}{\cancel{l_1}}$

$\theta = \frac{1}{2}\frac{XE l_1}{m v^2}$ —(1)

angle of dispersion of the beam $\Big\}$ $d\theta = -\cancel{2}\,\frac{1}{\cancel{2}}\frac{XEl_1}{m}(v^{-3}dv)$

$d\theta = -\frac{XEl_1}{m}\frac{dv}{v^3}$ —(2)

$= -\left(\frac{XEl_1}{mv^2}\right)\frac{dv}{v}$

$d\theta = -\frac{2\theta\, dv}{v}$

$\therefore \frac{d\theta}{\theta} = -\frac{2dv}{v}$ —(3)

IIIly if d_2 = displacement of the ion from its path due to the magnetic field of strength B. and l_2 is the length of path of the ion in the magnetic field

$d_2 = \frac{1}{2}at^2$

$a = \frac{F}{m}$

$t = \frac{dis}{v_2}$

$d_2 = \frac{1}{2}\left(\frac{F}{m}\right)\left(\frac{d_2}{v_2}\right)^2$

$$d_2 = \frac{1}{2}\frac{Bev_2}{M}\frac{l_2^2}{v_2^2}$$

deviation produced $= \frac{1}{2}\frac{Bev_2}{m}\frac{l_2^2}{v_2^2}$

$$\phi = \frac{d_2}{l_2} = \frac{\frac{1}{2}\frac{Bev_2}{m}\frac{l_2^2}{v_2^2}}{l_2}$$

$$\phi = \frac{1}{2}\frac{Be}{m}\frac{l_1}{v} \quad \text{--- (4)}$$

$$d\phi = \frac{1}{2}\frac{Be}{m} l_1 \left[-1\, v^{-2} dv\right]$$

$$d\phi = -\frac{1}{2}\frac{Be\, l_1\, dv}{mv^2}$$

$$d\phi = -\left(\frac{\frac{1}{2}Bel_1}{mv}\right)\frac{dv}{v} \quad \text{--- (5)}$$

$$d\phi = -\phi\frac{dv}{v}$$

$$\frac{d\phi}{\phi} = -\frac{dv}{v} \quad \text{--- (6)}$$

comparing (3) + (6)

$$\frac{d\theta}{\theta} = \frac{2\,d\phi}{\phi} \quad \text{or} \quad \frac{d\phi}{d\theta} = \frac{\phi}{2\theta} \quad \text{--- (7)}$$

Let $a = OR$ = distance between two fields. The width of the selected group of ions at $R = a\,d\theta$. If there had been no magnetic field, the width of the group after travelling further distance b would be $(a+b)\,d\theta$. The magnetic field produces a convergence $d\phi$ and brings the group of ions by a focus at a distance $RF = b$

The condition for focusing is

$$(a+b)\,d\theta = b\,d\phi$$

$$\frac{a+b}{b} = \frac{d\phi}{d\theta}$$

From eqn (1)

$$\frac{a+b}{b} = \frac{\phi}{2\theta} \qquad \frac{a}{b}+1 = \frac{\phi}{2\theta}$$

$$\frac{a}{b} = \frac{\phi}{2\theta} - 1 \Rightarrow \frac{\phi - 2\theta}{2\theta}$$

$$\frac{a}{b} = \frac{\phi - 2\theta}{2\theta}$$

$$\frac{b}{a} = \frac{2\theta}{\phi - 2\theta} \quad \text{—— (2)}$$

This expression represents a straight line drawn from O, making an angle 2θ with the direction of the beam deviated by the electric field. If $\phi = 2\theta$, the rays do not converge (since $b = \infty$). However, if $\phi = 4\theta$, $b = a$ and the convergent beam can be easily photographed. The ions are focused at F such that $OR = RF$.

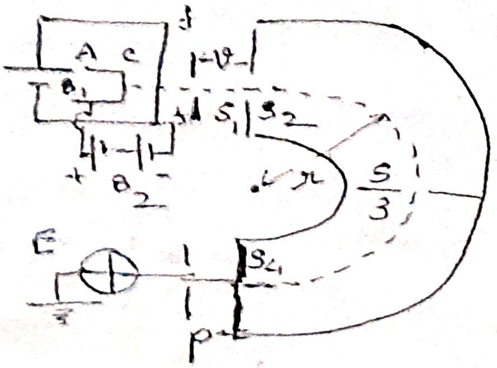
advantages

1) particles having same E/M value are focused at a single point. Thomson method they are spread out to a parabola. ∴ intensity of line on to photographic plate are large.

2. while in thomson is weak.

Dempster's Mass spectrograph.

The experimental arrangement are shown



The anode is a metal cylinder A with its front surface C coated with a salt of the element under test and heated electrically. A filament, if electrically heated by the battery B_1 emits electrons. By maintaing the filament at a PD of about 50 volts with respect to A by using another battery B_2, the electrons are made to bombard the heated salt with the result that the anode emits positively charged ions of the element. These ions are collimated into a narrow beam by the slit S_1. Then the positive ions are accelerated towards the slit S_2 by a variable P.D V maintained between S_1 and S_2.

We know that when ions of mass M and charge e are accelerated through a P.D. V, they acquire a velocity v given by

$$\frac{1}{2} M v^2 = eV \quad \text{or} \quad v = \sqrt{\frac{2eV}{M}} \quad \text{— (1)}$$

The velocity of the ions v is very large compared with their initial speed of emission. The ions then enter a vacuum chamber G. On entering the space G, the ions are subjected to a magnetic field of flux density B is directed at right angles to the plane of the diagram. The ions are deflected through a semicircle towards the slit S_1 defined by the three slits S_1, S_2 and S_3. The magnetic field of flux density B makes an ion of mass M entering with the velocity v a transverse a circular path of radius r given by

$$Be\cancel{v} = \frac{Mv^2}{r}$$

$$r = Mv/Be \quad \text{———} \quad (2)$$

from (1)

$$\therefore \quad r = \frac{M}{Be}\sqrt{\frac{2eV}{M}}$$

$$= \frac{\sqrt{m}\,\cancel{\sqrt{m}}}{B\sqrt{e}\,\cancel{\sqrt{e}}}\sqrt{\frac{2\cancel{e}v}{\cancel{M}}}$$

$$r = \sqrt{\frac{2mv}{B^2e}} \quad = r^2 = \frac{2mv}{B^2e}$$

$$\frac{e}{m} = \frac{2v}{B^2r^2} \quad \text{———} \quad (3)$$

From this particular value of v only ions with specific charge given by eqn (3) are collected by the electrode P and the current is registered by the electrometer E. From the equation $\left(\frac{2MV}{B^2e}\right)^{1/2}$ = r it is clear that if $\left(\frac{M}{e}\right)\cdot\frac{V}{B^2}$ is kept constant the radius r of the path will remain unchanged